Philos Stud
DOI 10.1007/s11098-012-0030-5

What are groups?

Katherine Ritchie

Abstract In this paper I argue for a view of groups, things like teams, committees, clubs and courts. I begin by examining features all groups seem to share. I formulate a list of six features of groups that serve as criteria any adequate theory of groups must capture. Next, I examine four of the most prominent views of groups currently on offer—that groups are non-singular pluralities, fusions, aggregates and sets. I argue that each fails to capture one or more of the criteria. Last, I develop a view of groups as realizations of structures. The view has two components. First, groups are entities with structure. Second, since groups are concreta, they exist only when a group structure is realized. A structure is realized when each of its functionally defined nodes or places are occupied. I show how such a view captures the six criteria for groups, which no other view of groups adequately does, while offering a substantive answer to the question, "What are groups?"

Keywords Metaphysics of social science · Ontology · Structuralism

> A class…, in one sense at least, is distinct from the whole composed of its terms, for the latter is only and essentially one, while the former, where it has many terms, is… the very kind of object of which many is to be asserted.
>
> —Bertrand Russell (1903, p. 69)

Common sense, natural language data and the practices of governments, sports fans and sociologists give us reason to think that social groups, things like clubs, committees and teams, exist. Here, I will not canvass evidence in favor of the

K. Ritchie (✉)
University of Texas at Austin, Austin, TX, USA
e-mail: kcritchie@gmail.com

Published online: 10 October 2012

existence of groups, but will operate under the supposition that groups exist. Instead, I undertake the task of examining the nature of groups. I begin in Sect. 1 by formulating a list of criteria that any adequate theory of groups must capture. Then in Sects. 2–5, I turn to views of groups which have been proposed. These include the views that groups are non-singular pluralities, fusions, aggregates and sets.[1] I argue that each fails to capture one or more of the criteria. Last, I develop a novel view of groups as realizations of structures, which can capture all of the criteria while offering a substantive answer to the question, "What are groups?". Russell drew a distinction between a class as one and a class as many. In the same vein, we might think there is a distinction between a group as one and as many. Groups as one have members. Given this, we might hope that a view of groups as one deliver a way to determine a group as many.[2] We will see that the view of groups as one I propose also delivers a view of a group as many.

1 Features of groups

Before examining any view of groups, it will be useful to set forth features all groups seem to have in common. Any adequate view of groups must allow for groups to have all of these features. First, groups can have different members at different times. For instance, a team might grow (when a new player is acquired) or shrink (when a player is traded). Second, groups can have different members across worlds. In the actual world, Edolphus Towns is a member of the Committee on Oversight and Government Reform, but in another possible world the Committee might fail to have Towns as a member. Third, groups, like other objects, can exist at one time without existing at every time. For example, before basketball was invented there were no basketball teams, so no basketball team has existed at every time. Groups are not (or at least need not be) eternal beings. Fourth, groups, like other ordinary objects, might exist at one world without existing at every world. That is to say, groups are not necessary beings. For instance, in the actual world the Committee on Oversight and Government Reform exists, but in a world without any formally organized governments it fails to exist. Fifth, as well as being located in time, groups are (or can be) located in space.[3] A Committee might be meeting in Washington, while a chess club might be meeting at a high school in Minneapolis. Last, there can be coincident groups of the same basic kind. For instance, the same individuals might make up the chess club and the nature club. However, even with identical extensions at a time the two clubs fail to be identical. In such a situation it

[1] Gabriel Uzquiano (2004) has argued that groups are sui generis entities. For reasons of space I will not address his view here. The four views I examine and the view I develop here all take it upon themselves to give a substantive answer to the question regarding the nature of groups. If a view which gives a substantive answer to that question can be given, I take that to be a mark in favor of it over the view that groups are sui generis.

[2] At least relative to a time and world.

[3] In some video games one can form teams. Depending on what one says about the metaphysics of virtual characters, one might not take such a team to be located in space. However, the most common examples of teams and other groups are located in space.

would be neither inappropriate nor false for a student to say that she is in two clubs—the chess club and the nature club.[4]

One might think that a further important feature to capture has to do with groups having more than one member. Teams, committees and clubs all seem to be things made up of many members. While groups usually have multiple members, I do not take this to be a requirement due to cases like the following. Suppose that the United States is in turmoil. One senator resigns from office, then another and another until finally all but one senator has resigned. The Senate, if it exists at all, is a group made up of one member. A bit later, new elections are held and new senators are elected. The Senate grows and is, seemingly again, a many-membered group. While one could argue that the Senate before the mass exodus is a distinct entity from the Senate after it is quite natural to say that the Senate shrunk and then grew.[5] I will not argue for either view here. Instead, I will leave out a criterion requiring groups to be many-membered to accommodate both views of the case. Further, the view of groups I develop is able to explain why groups often have many members.

While I do not take the criteria discussed above to exhaust the features of groups I do take them to capture important features that groups share. To summarize, the features groups share and which I take to be criteria an adequate view must capture are:

(1) *Members–Times* Groups can have different members at different times
(2) *Members–Worlds* Groups can have different members across worlds
(3) *Existence–Times* Groups can exist at one time without existing at every time
(4) *Existence–Worlds* Groups can exist at one world without existing at every world
(5) *Space* Groups are (or can be) located in space
(6) *Coincidence* Groups of the same basic kind can be extensionally coincident and non-identical

In the next four sections I examine prominent views of groups that have been developed. I begin each section by laying out a view. I then assess whether it captures the six features of groups given above. Before turning to these discussions I examine how group composition relates to material composition more generally.

For an examination of group composition in particular rather than material composition in general to be warranted one might argue that groups must have some features which distinguish them from tables, trees and other material objects. I have argued that groups share the six criteria set out above. If other objects do not share these features, this would mark a difference between groups and other wholes. At least the first five criteria seem to be satisfied by object generally. Most take it that

[4] This criterion goes against the view that two things cannot be in the same place at the same time. For example, David Wiggins notes that "it a truism frequently called in evidence and confidently relied upon in philosophy that two things cannot be in the same place at the same time" (Wiggins 1968, p. 90). I take it that those who take this to be a truism for all things have failed to adequately examine the properties of groups, for groups seem to clearly violate this rule.

[5] I thank an anonymous referee at Philosophical Studies for bringing a case structurally similar to this one to my attention.

tables and trees allow for variations in parts across times or worlds, are neither necessary nor eternal and take up space. However, the sixth criterion, that there can be non-identical coincident groups of the same basic kind, might mark a difference between groups and other objects. Could there be two non-identical but coincident tables? Or two completely overlapping but distinct Oak trees? While there can certainly be two coincident and non-identical clubs or teams, tables and trees seem different. Of course some hold that there might be coincident objects. For example, Kit Fine (1999) argues that a tree and the wood it is made of are necessarily coincident, but non-identical. He states that the wood and the tree are of different sorts, so even on a view that allows many coincident entities, coincident entities of the same basic kind seem to be ruled out. The Coincidence criterion marks one difference between groups and other wholes.

Social groups can also be distinguished from other objects by another feature—member intentions. The parts that make up a table do not intend to make up a table. In fact, they have no intentions at all. Similarly, while a person might have intentions, the parts that make up a person fail to have intentions, so they cannot jointly intend to compose a person. Social groups are different. The members of a team intend to form a team. A club is formed when individuals join together with the intention of forming a club. Even groups created by fiat involve intention. Suppose that President Obama decides to create a committee composed of the Chief Justice, the Majority and Minority Leaders of the House of Representatives, and the Majority and Minority Leaders of the Senate. Obama intends for these five individuals to form a new committee. Since the individual who is Chief Justice and the individual who is the Majority Leader of the House and so on have joined (through their intentions) groups which are part of a government that recognizes that President Obama has the power to form committees, one might hold that the new committee is formed through Obama's intention. Alternatively, one might hold that in addition to Obama's intention, the members must also intend to form the committee. Either way, intentions are part of what determines whether some things form a social group. The views examined in the next four sections and developed in the final section might be taken to be views of material objects generally. In order to draw a distinction between social groups and other material objects, a necessary condition that group members or others in suitable positions have group-forming intentions might be added to any of the views discussed in the next five sections. Next, I turn to the first view of groups—that groups are non-singular pluralities.

2 Groups are non-singular pluralities?

A proponent of a view of groups as non-singular pluralities argues that groups just are their many members taken together, but not joined together. On such a view groups are nothing over and above individuals. On this picture there is no group as one, rather there is only a group as many. A proponent of the view that groups are non-singular pluralities might say "The team played a game" or "A committee met to discuss judicial reform," but 'the team' and 'a committee' are to be understood only as about some many individuals, not as further entities.

Let's look at an example of how such a view might work. Instead of the truth of "A committee met to discuss judicial reform" requiring an entity to have done something, it requires that many entities (perhaps suitably arranged) have done something. For instance, it might require that the individuals arranged committee-wise met and discussed judicial reform. Since being arranged committee-wise does not require that there is anything that is a committee, but only that certain relations hold, there is no object that is required in addition to the individuals.

Before addressing whether this view is able to uphold the desiderata set forth in §1 I address a worry. One might think that including the non-singular pluralities view of groups is odd given that we began with the supposition that there are groups. The claim that groups just are many individuals might sound like a claim that while we talk (think, act…) as if there are groups, really there are no groups, but only individuals. However, the proponent of non-singular pluralities view can talk as we did at the beginning of our investigation. She might accept the claim that there are groups, but understand it as requiring that there are individuals arranged group-wise. While such a position might not fit the spirit of the supposition with which we began our examination it is near enough to be appropriate to include.

Let's now turn to how the non-singular pluralities view of groups handles the criteria laid out earlier. I begin by looking at those that it does uphold. First, the proponent of the non-singular pluralities view is able to maintain Existence–Time, the claim that groups can exist at one time without existing at every time. On this view groups are nothing but many individuals. Since individuals can exist at one time without existing at every time, groups too can exist at one without existing at every time.

Similarly, the pluralities view can accommodate the Existence–World criterion. Since individuals might exist at some world without existing at every world, groups, which are just many individuals, can also exist at some world without existing at every world. Further, as long as individuals are located in space, a proponent of the pluralities view can hold that groups are located in space, thereby satisfying Space.

Of the remaining three criteria for groups, two are not satisfied on the view in question. Whether the third is satisfied is controversial. Let's start with the more controversial case. We took it that there could be non-identical coincident groups. On a straightforward understanding it seems that the non-singular pluralist cannot accommodate Coincidence. Suppose that some people, Ted, Joe, Angelika and Terry, make up a chess club and these same people make up a nature club. The individuals Ted, Joe, Angelika and Terry are identical with themselves. Since the chess club just is those four people and the nature club is also just those four people, the chess club is the nature club. The 'two' clubs are really one club described in two ways.

One might take the non-singular pluralist to have a viable response. She might argue that her view is not simply that a group is some individuals, but that a group is some individuals arranged in a suitable way. Some individuals might be arranged chess-club-wise and nature-club-wise. Since the arrangements are different, the groups, while coincident, are not identical. So, there can be coincident groups.

In utilizing arrangements one might worry that the non-singular pluralist is pitted with a position that is not nearly as nominalistic as the view originally seemed. It seems, one might argue, that the non-singular pluralist has been forced to reify arrangements. A group is no longer merely some individuals. Instead it is some individuals combined with an arrangement. If this is correct, attempting to capture Coincidence leads to a rejection of the non-singular pluralist view. Here, I will not further examine whether adding arrangements into the picture leads to a rejection of the non-singular pluralist view. Doing so would require an examination of the metaphysics of arrangements which is far beyond the scope of this paper. However, we will see that the non-singular pluralist has other problems which give us reason to abandon it.

We can now turn to an examination of the two criteria that the non-singular pluralities view definitively cannot handle. First, on this view groups cannot have different members across times, so Members–Times is not met. A group is just some individuals (perhaps suitably arranged). Individuals d, e and f are identical to individuals a, b and c if, and only if, d is identical to one of a, b or c, e is identical to one of a, b or c and f is identical to one of a, b or c and each of a, b and c is identical to one of d, e or f. Suppose that a group, G, is a, b and c arranged committee-wise. If, at a later time, one attempted to add an individual not identical to a, b or c to G, a new group would come into existence. This is represented in the following argument:

1. G is a, b and c (suitably arranged)
2. Individuals a, b and c are not identical to the individuals a, b, c and d.
3. So, G is not a, b, c and d (suitably arranged).

Members–Times is not met.

Instead of trying to add d to G at a different time, one might try to add d to G at a different world. The same sort of argument can be employed to show that the non-singular pluralities view of groups fails to capture Members–Worlds, the criterion that groups can have different members at different times. The chart below summarizes how the non-singularist pluralist handles the criteria for groups.

Non-Singular Pluralities	Members–Times	Members–Worlds	Existence–Times	Existence–Worlds	Space	Coincidence
Captured?	No	No	Yes	Yes	Yes	Maybe, if arrangements can be included

The non-singular pluralist fails to accommodate that groups can change members across times and worlds. The view should be rejected. Groups should be reified in a way in which the non-singular pluralist refuses to allow. In the remaining sections I consider only views which take groups as one seriously.

3 Groups are fusions?

A proponent of the view that groups are fusions holds that members of a class fuse to form a whole. Theodore Sider, a proponent of the fusion view, defines a fusion as:

> x is a fusion at a time, t, of a class, S, iff (1) every member of S is a part of x at t, and (2) every part of x at t overlaps-at-t some member of S (2001, p. 58).

A proponent of fusion relies on a parthood-at-a-time relation. This relation might be defined as:

> x is a part of y at t iff x and y each exist at t, and x at t is a part of y at t.[6]

Parthood is a transitive relation, so anything that is part of the fusion, or a part of a part of the fusion, and so on is a part of the fusion. The identity conditions for fusions are usually given extensionally. Given this, and since parthood is relativized to times we might take the primary notion of identity to be relativized to time as in:

> Two fusions z and y are identical at a time, t, if, and only if, for any x, x is a part of z at t, if, and only if, x is a part of y at t.

Unrelativized identity might then be taken to be identity at every time at which either z or y exists. The following table shows how the fusionist view stands on the criteria for groups.

Fusions	Members–Times	Members–Worlds	Existence–Times	Existence–Worlds	Space	Coincidence
Captured?	Yes—because the parthood relation is relativized to times	If one appeals to counterpart theory, Yes	Yes—because a fusion exists only when some part of it exist	Yes—if none of a fusion's (counter)parts exist at a world, the fusion does not exist at that world	Yes—because fusions exist where their members exist	No

The fusionist has trouble capturing Coincidence. The fusionist cannot capture Coincidence since the identity conditions of groups when they are taken to be fusions are extensional. Take the following unrelativized identification principle for fusions: two fusions, z and y, are identical if, and only if, for any x, x is a part of z if, and only if, x is a part of y. Given this principle, Coincidence cannot be satisfied. If groups are fusions, then two extensionally equivalent groups are identical. For example if the chess club and the nature club have all of the same parts at time t, they are a single club at time t. So, on this view it is strictly false for a student to say that she is in two clubs when the club memberships overlap completely.

One might argue that while strictly a student is in one club at a time when all the members of the chess club are all and only the members of the nature club, the clubs themselves are distinct due to times at which they differ in membership. While such a line might be promising for some cases, there are other cases for which it will fail. For instance, suppose that a new committee is formed which includes all and only the Supreme Court Justices. The new committee and the Supreme Court will have all and only the same members at every time at which they exist. So, on the fusionist view they will be one group. However it is certainly possible for such a committee to exist. It is not a court so it is not identical to the Supreme Court. Yet on the fusionist view such a committee and the Supreme Court are identified.

[6] This is similar to the explication Sider gives of parthood at a time.

The fusionist might appeal to counterpart theory[7] to try to capture Coincidence. For example, suppose that in another world the chess/nature club of the actual world has two counterparts g1 and g2. G1 has a part, d, that g2 does not have, but otherwise g1 and g2 overlap. Suppose that one takes g1 to be the nature club and g2 to be the chess club. Since g1 has a part that g2 fails to have, it turns out that it could have been the case that something was a part of the nature club without being a part of the chess club.

Contra the fusionist, one might argue that g1 is a counterpart of the chess club as much as it is counterpart of the nature club. So, an appeal to counterparts shows only that there could be two clubs that are both identical to a single actual club. Counterpart theory, such an objector might argue, fails to show that only the nature club, and not the chess club, could have had some part, d. Since these two possible clubs are not identical they need not have all parts in common, but given that the nature club and the chess club are actually co-extensional a counterpart of one is a counterpart of the other.

To reject this conclusion the fusionist might argue that something is a counterpart of an actual thing qua a sortal. For instance, something might be a counterpart of a group qua nature club or qua chess club. These counterpart relations could be taken to have different standards of application even in a single context. Since the relations differ, they need not stand or fall together. In the example above, the actual club might bear the counterpart qua nature club relation to g1 (and not g2) and the counterpart qua chess club relation to g2 (and not g1). Since g1 has a part that g2 does not have and since g1 is a nature club counterpart of the actual nature/chess club while g2 is not a nature club counterpart of the actual nature/chess club, the chess club might not have been co-extensional with the nature club.

Adding qua-counterpart relations does not capture genuinely coincident groups. To capture Coincidence we wanted some individuals at a time to form two distinct groups, in our example a nature club and a chess club. By appealing to counterpart theory, the fusionist argued that *that* group might have many complex modal properties. It might have the property 'being counterpart related to g1 qua nature club relation.' Further, it might have the property 'being counterpart related to g2 qua the chess club relation.' These are two properties that the one actual chess club/ nature club fusion possesses. Having more than one modal property, even qua some sortal, does not allow a single actual fusion to satisfy Coincidence. The fusionist is unable to adequately handle Coincidence. Since we took it that there can be coincident but non-identical groups, the view that groups are fusions should be rejected. Next I turn to the view that groups are aggregates.

4 Groups are aggregates?

Tyler Burge (1977) developed a theory of aggregates based on set-theoretic principles. Burge restricts his theory to the first-order, allowing only sets with

[7] In counterpart theory individuals are usually taken to be world-bound. An individual has many counterparts according to different relations. These relations ground the truth of counterfactual claims. See Lewis (1986).

individuals as members. He excludes the null set and identifies singleton sets with their members. The 'member of' relation is replaced by the 'member–component of' relation, which is reflexive and intransitive. As Burge develops the theory, what counts as a member–component of an aggregate is determined by a plural construction like "the stars in the Pleaides galactic cluster." Here, the Pleaides galactic cluster's member–components are stars. The gases and molecules and … that make up the stars are not member–components of the galactic cluster. Since we are concerned with things often referred to without the use of a plural (e.g., in 'the Supreme Court'), language is a less clear guide to member–components. However, a privileged sort of member–component is usually easy to find. The Supreme Court has justices as member–components. A high school chess club has students as member–components. Burge uses the member–component relation to define aggregates:

> x is an aggregate if, and only if, for some z and some y (z is a member-component of x & y is a member-component of x & z is not identical to y) (1977, p. 100).

Two aggregates, z and y, are identical if, and only if, any individual x is a member–component of z if, and only if, it is a member–component of y. An aggregate can sustain alterations in parts only if the parts are not member–components. That is, an aggregate might alter in the parts of its member–components, but not in its member–components themselves. An aggregate persists if it retains all of its member–components and gains no new member–components. Burge takes aggregates to be located in space and time. They are located where their member–components are located. They come into and pass out of existence along with their member–components. The following table shows how the aggregate view of groups handles the criteria.

Aggregates	Members–Times	Members–Worlds	Existence–Times	Existence–Worlds	Space	Coincidence
Captured?	No	If one appeals to counterpart theory, Yes	Yes—because it exists only when its member–components exist	Yes—because it exists only at worlds in which its member–components exist	Yes	If what counts as a member–component is the same, No

The aggregate view has trouble with three of the criteria. I begin by examining Members–Times. Burge allows for aggregates to sustain alterations in parts, but not in member–components. If Members–Times is understood as requiring only that some change in part-hood is allowed, the aggregate view holds up. However, it was meant to capture the ability for a team or committee to gain or lose not only molecules, but also players and committee–members. The view that groups are aggregates does not allow for Members–Times to be upheld on the robust understanding that was meant.

Given that an aggregate cannot lose a member–component at a world, one might conclude that an aggregate cannot vary in member–components across worlds, thereby failing to satisfy Members–Worlds. However, like the fusionist one might appeal to counter-part theory. One would then allow for an aggregate to have counterparts with different member–components.

Last, the aggregate view of groups fails to capture Coincidence in many cases. Burge takes two aggregates, and so, on the view in question, two groups, to be identical if they have all and only the same member–components. In the chess club and nature club example above, each have the same member–components. So, on the groups are aggregates view, "they" are a single club.[8] Like the fusionist, one might here too appeal to counterpart theory to try to avoid rejecting Coincidence. However, the moves available here will be similar to those already canvassed and shown to ultimately be unhelpful options for the fusionist. Since they would be put to the same use by the proponent of the aggregate view here, they will also be unhelpful. So, I will not rehash them here. Next, I examine the view that groups are sets.

5 Groups are sets?

A simple view of groups as sets runs into problems straightaway. Sets cannot change members across times and worlds, while groups can. Sets are not and groups are located in space. Coincident sets are and coincident groups are not identified. Nikk Effingham (2010) recently developed a more sophisticated version of the set view of groups which attempts to handle these problems. On his view a group is defined as follows where 'member$_S$' is set membership:

> g is a set with only ordered pairs as member$_S$; (b) every possible world is the first member of exactly one ordered pair that is a member$_S$ of g; and (c) the second member of each such ordered pair is itself a set of ordered pairs such that (i) every instant is the first member$_S$ of exactly one of those latter ordered pairs and (ii) the second member$_S$ of each of the latter ordered pairs is either the empty set or a set of individuals (2010, p. 259).

On this view groups are sets of tuples with worlds as their first member and sets of tuples of times and either sets of individuals or the empty set as their second member. Group membership, written as 'member$_G$', is defined partially in terms of set membership as follows:

> x is a member$_G$ of g at t at world w just in case (i) g is a group; (ii) g has the ordered pair z as a member$_S$ where (iii) z has w as the first member$_S$ and (iv) z has z* as its second member$_S$; (v) z* has as an ordered pair y as a member$_S$ where (vi) y has t as its first member$_S$ and (vi) x is a member$_S$ of the second member$_S$ of y (2010, p. 259).

[8] Coincidence can be captured when two aggregates have different member–components. For instance, suppose there is an aggregate of committees and an aggregate of senators. All and only the individuals in the aggregate of senators are in the aggregate of committees. However, the aggregate of committees has committees, not senators, as member–components. So, the two groups are distinct.

The table below shows how the sophisticated setist handles the criteria.

Sets	Members–Times	Members–Worlds	Existence–Times	Existence–Worlds	Space	Coincidence
Captured?	Yes	Yes	Maybe	Maybe	Maybe, but only if one takes some sets to be located	No, but it may capture it in spirit

The proponent of a sophisticated set view of groups has trouble with four of the criteria. First, sets are usually taken to be abstract. If one holds this standard view about sets, Space is not captured. To capture Space a setist must argue that (at least some) sets are concrete.

Second, the sophisticated set view does not capture Coincidence since set identity obeys the following: for any set, s, and any set, s', if s has all and only the same members as s', then s is identical to s'. Given this principle co-extensional sets are identified. However, the sophisticated setist can hold that two actually coincident groups are not identical as long as the two differ in members across worlds. The most straightforward cases we have considered, like the nature club and the chess club, will be such that they will vary in members across worlds. So, perhaps the setist captures the spirit of Coincidence by allowing for two groups to be distinct when possibly, but not when necessarily, coincident.

Since groups are identified with sets which include every world and time, the setist appears to take groups to exist at every world and time, thus violating Existence–Times and Existence–Worlds. A setist might argue that while sets exist necessarily and eternally, groups exist only when and where the members of the second member of their ordered pairs beginning with instants exist. If this is the case, whenever a time is paired with the empty set, the group does not exist. If all of the instants of the ordered pairs associated with some world, w', have the empty set as their second member, the group does not exist at that world. With these modifications, a group might exist at only some times and worlds.

While arguing in this way allows for Existence–Times and Existence–Worlds to be captured it is ad hoc. Why is it the second member of the ordered pair that begins with an instant which determines whether the group exists at a time or world? The setist identifies groups with a set whose precise construction appears arbitrary. Benacerraf gave a worry of this sort against the view that numbers should be identified with sets. Benacerraf (1973) argued that there are sets that are equally good candidates with which to identify a given number. Since choosing to identify, for example, the number 2 with a set constructed according to Zermelo-Fraenkel set theory rather than, say, von Neumann set theory is arbitrary, 2 should not be identified with either set or with any of the other candidates on offer. Similarly, there is no reason why groups are identified with sets with ordered pairs with instants as their first member rather than as their second member. Since identifying groups with one candidate set rather than another would be arbitrary, groups should not be identified with any set.

Effingham acknowledges that a Benacerraf problem exists for the setist. He suggests two responses which take the worry seriously. First, he suggests identifying groups with sets which bear the ‘same group as’ relation to one another. Sets constructed as Effingham’s describes and those which are constructed in the same way, but which include ordered pairs with a set of group members as the first member and an instant as second member bear the ‘same group as’ relation to one another. If the setist opts for this response Existence–Times and Existence–Worlds cannot be captured. To capture these criteria a claim like the following is needed: a group is located only where the second members of its ordered pairs which have instants as their first members are located. If groups are identified with classes of sets, such a fix is no longer available.

Second, Effingham suggests, but does not develop, that one might appeal to a structuralist view of groups to avoid an arbitrariness worry. He concedes that in opting for this route one no longer maintains that groups are sets. So, the setist cannot avoid a Benacerraf-style problem in this way. In the next section I develop a structuralist view of groups.

The four views assessed each fail to capture one or more of the criteria. In the next section I present a new view of groups which captures the six criteria. Further, it captures a distinctive feature of groups—that they are entities with structure—that is not captured by the views just canvassed.

6 Groups are realizations of structures

The structuralist view of groups I propose and sketch here has two components. The first captures the importance of the maintenance of a group’s structural organization to a group’s persistence. The second captures the requirement that a group must be made up of things. I discuss these in turn.

One component of a group is its structural organization. The structure of a group can be represented with nodes (or places) and edges connecting nodes to other nodes. The edges of a structure capture the relations that hold between nodes. Since all members of a group are related to some degree, each node in structure S is connected to every other node in S. Functional relations connect nodes in group structures. Some of these functional relations might be hierarchical in nature. Hierarchical relations capture the “order of command” or power relations between nodes. Such relations might be represented by directed edges.[9]

Many non-hierarchical functional relations might hold between nodes. For instance, in the baseball team structure the node labeled ‘pitcher’ is related to that labeled ‘catcher’ by the pitch-ball-to relation. Similarly catcher[10] is related to pitcher by the return-ball-to relation. Some functional relations which appear via description to be the same differ upon closer examination. For instance, the node

[9] For views on the nature of structures see Shapiro (1997), Resnik (1997), Hellman (1989) and Putnam (1967)

[10] From here I use “catcher,” “pitcher” and the like to mean “the node labeled ‘catcher” and “the node labeled ‘pitcher,’” respectively. I still occasionally use the longer “the node labeled ‘N’” when it makes the discussion more clear.

labeled ‘shortstop’ is related to pitcher via the return-ball-to relation since that is a relation that might hold between individuals occupying those nodes. However, it seems to hold more strongly between pitcher and catcher than between shortstop and catcher. In the first it holds more frequently and seems more integral to the way the baseball team functions than in the second.

To represent this seeming difference we might take one of three options. First, edges in a structure might be weighted. A more heavily weighted edge marks a relation which holds more strongly. On this option, structures are fine grained as they include not just places and relations, but strengths with which relations hold. The weights might be represented by a number or might be specified only relative to other relations in the structure. On the first option the weight of the ‘return-ball-to’ relation as it holds between pitcher and catcher might be represented by .8 and as .4 between shortstop and pitcher. Alternatively, it might be represented as holding more strongly between pitcher and catcher than between pitcher and shortstop without specifying a precise weight, but rather through a partial ordering of relations. The ‘return-ball-to’ relation as it relates catcher and pitcher will be higher in the ordering than it as it relates shortstop and pitcher.

Second, one might hold that structures are made up of only nodes and relations. To capture the difference between the return-ball-to relation as it holds between a pitcher and a catcher and as it holds between a shortstop and a catcher one might take ‘a group’s structure’ to pick out a vaguely delimited class of structures. The structures all share some features in common, but might include slightly different relations. A group exists when some suitable number of the class of structures is realized.

Third, one might argue that instead of ‘a group’s structure’ picking out a vaguely delimited class of structures the reference of ‘a group’ is vague. There are many different groups. Each is a realization of some structure which is made up of nodes and relations. The structures share many similarities, but include some differences, perhaps in their relations.[11] I will not settle on one of these options here. Instead, the view that groups are realizations of structures might be developed in accordance with any of the three.

In addition to intra-structural functional relations, functional relations hold inter-structure when a structure is (or some structures are) embedded in a larger structure. For instance, when engaged in playing a game two teams are a part of a game structure. Relations hold between, for example, a pitcher on Team 1 and a batter on Team 2. The teams might also be part of league or division structures. Inter- and intra-structural roles wholly define the nodes in a structure, as nodes are defined only in terms of structural or functional relations.

Nodes in a structure might have one or more than one satisfier at a time. Some node might be marked to allow only one satisfier at a time (e.g., the ‘Chief Justice’ node) while some other might allow one or many satisfiers (e.g., the ‘treasurer’ node). By allowing for a varying numbers of satisfiers of a node, a group can alter in size without a structural change. A node that was once occupied by only a single satisfier, might later be occupied by more than one satisfier. To capture relations

[11] I thank an anonymous reviewer at Philosophical Studies for comments on the grain of structures and for suggesting the third option discussed here.

between individuals who share a node in a realization of a structure, nodes must bear reflexive relations to themselves. In some structure this will be the 'has-same-functional-role' relation, in others it might be something more specific.

Next, I turn to the second part of the structuralist view of groups, explaining how members of a group at a time and world relate to the structure of a group. When a group exists it has both a structure and some members which occupy the nodes in the structure. Some things are members of group G with structure S at time t and world w just in case they jointly realize S. Some things jointly realize a structure if, and only if, each occupies a node (or some nodes) in the structure and every node in the structure is occupied by one or more of the things. To occupy a node is to stand in the relations required by the node. It is only through the efforts or actions of the many members of a group that all of the nodes in the structure are filled and that the structure of the group is realized.

Occupying a node requires the satisfaction of two conditions. First, something occupies or some things occupy a node in a structure S if, and only if, every node in S is occupied. Second, the occupier(s) of a node, n, must stand in the relations required by n to the other things occupying nodes in S. If these two conditions are met, X[12] occupy a node in S.

One thing might occupy the role of more than one node. For instance, someone might be both Secretary and Treasurer of a club. By allowing for a single thing to occupy more than one node, a group structure might be realized by a single thing. While this is possible, it is uncommon due to the sorts of structures most groups have. Most group structures include at least one irreflexive relation. For example, in the baseball group structure the relation 'return-ball-to' is irreflexive. All hierarchical relations like has-more-power-than are irreflexive. Committees with bylaws based on Robert's Rules of Order will include the seconds-a-motion-introduced-by relation, which is irreflexive. More generally, many groups require collaboration between members. Since collaboration is not something one can do with oneself (i.e., collaborates-with is an irreflexive relation), many groups require more than one member. Since irreflexive relations cannot relate a thing to itself, a realization of a structure with an irreflexive relation requires at least two entities.

Once a group structure is realized a group, G, exists. The persistence of G requires the continuity of the realization of S. That is, occupiers of the nodes of S which form the realization of group G (i.e., members of G) must continue to bear the functional relations required for a realization S. Persistence of G allows for alterations in members given the following understanding of group membership:

> Some thing, x, is a member of group G with structure S at time t and world w if, and only if, x occupies a node(s) of S and is functionally related (in the ways required by S) to other occupiers of nodes in S of G.

If something, x, comes to occupy a node in S at t' and at w, it is then related to other occupiers of nodes of S in its realization as group G and is a member of G at time t' at w. If something, y, which was a member of G at t at w, fails at time t' to bear

[12] Here X is a plural variable. Standardly in plural logic, plural variables allow for something or somethings to be taken as argument. See, for example, Boolos (1984).

functional relations (in the ways required by S) to other occupiers of nodes of S, y ceases to be a member of G at t' at w. Given this group identity at a time and world can be defined as: A group g1 and a group g2 are identical at t at w if, and only if, (1) g1 and g2 have the same members at t at w, and (2) g1 and g2 include all the same intra- and inter-structural relations holding between the same individuals at t and w.

The view that groups are realizations of structures treats groups as one rather than many. A group is a realization structure. However, a view of groups as many is also delivered. What a group as many is is relative to a time and world. Some things count as the many things making up a group g at time t and world w just in case they are all and only the things that make up g at t. The things jointly "make up" g just in case they are all and only the things that occupy the nodes in the structure of g. The proposed view of groups as realizations of structures can capture the notion of a group as many and a group as one.

The view that groups are realizations of structures has advantages over its rivals. It captures all six criteria. Some thing might be a member of a group G only at some times and worlds. So, Members–Times and Members–Worlds are captured. A group, G, exists only at times when and worlds where there is a realization of its structure. So, Existence–Times and Existence–Worlds are captured. Groups are located where their realizations are. Groups are (usually) realized by physical entities. So groups are (usually) located in space. Last, two groups which have all the same members are not identified if there is any difference in structural relation. These structural differences might be inter- or intra-structural, so Coincidence is captured. The table below summarizes how the view that groups are realizations of structures handles the criteria.

Realizations of Structures	Members–Times	Members–Worlds	Existence–Times	Existence–Worlds	Space	Coincidence
Captured?	Yes	Yes	Yes	Yes	Yes	Yes

The view that groups are realizations of structures captures all six criteria while giving a substantive answer to the question, "What are groups?" Further it gives a neat picture of groups as many as well as groups as one. Of the views of groups currently on offer, the view that they are realizations of structures comes out on top.

Acknowledgments I would like to thank audiences at the 2011 meeting of the Pacific Division APA and at the University of Texas for valuable feedback on talks based on earlier versions of this paper. In particular, I would like to thank Josh Dever, Kendy Hess, Hans Kamp, Robert Koons, Mark Sainsbury, Nora Berenstain and an anonymous referee at Philosophical Studies for their comments and criticisms.

References

Benacerraff, P. (1973). Mathematical truth. *Journal of Philosophy, 70*(19), 661–679.

Boolos, G. (1984). To be is to be the value of a variable (or some values of some variables). *Journal of Philosophy, 81*, 430–450.

Burge, T. (1977). A theory of aggregates. *Nous, 11*, 97–117.

Effingham, N. (2010). The metaphysics of groups. *Philosophical Studies, 149*, 251–267.
Fine, K. (1999). Things and their parts. *Midwest Studies in Philosophy, XXIII*, 61–74.
Hellman, G. (1989). *Mathematics without numbers*. Oxford: Clarendon Press.
Lewis, D. (1986). *On the plurality of worlds*. Oxford: Blackwell.
Putnam, H. (1967). Mathematics without foundations . In: Benacerraf & Putnam (Eds.) (1983). Philosophy of mathematics: Selected readings (2^{nd} ed., pp. 295–311). Cambridge: Cambridge University Press.
Resnik, M. (1997). *Mathematics as a science of patterns*. Oxford: Clarendon Press.
Russell, B. (1903). *Principles of Mathematics*. Cambridge: Cambridge University Press.
Shapiro, S. (1997). *Philosophy of mathematics: Structure and ontology*. Oxford: Oxford University Press.
Sider, T. (2001). *Four-dimensionalism*. Oxford: Oxford University Press.
Uzquiano, G. (2004). The supreme court and the supreme court justices: A metaphysical puzzle. *Nous, 38*, 135–153.
Wiggins, D. (1968). On being in the same place at the same time. *Philosophical Review, 77*(1), 90–95.